Second-order linear homogeneous DE with constant coefficients:

$$\boxed{ay'' + by' + cy = 0}$$

Characteristic equation: $a\lambda^2 + b\lambda + c = 0$

Two distinct, real roots $\lambda_1 \neq \lambda_2$:

Fundamental solutions: $e^{\lambda_1 t}$, $e^{\lambda_2 t}$

General solution: $c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

Example: $y'' + 3y' + 2y = 0$

Characteristic equation: $\lambda^2 + 3\lambda + 2 = 0$

$(\lambda+1)(\lambda+2) = 0$

$\lambda = -1, -2$

Fundamental solutions: e^{-t}, e^{-2t}

General solution: $c_1 e^{-t} + c_2 e^{-2t}$

Repeated real root λ:

Fundamental solutions: $e^{\lambda t}$, $te^{\lambda t}$

General solution: $c_1 e^{\lambda t} + c_2 t e^{\lambda t}$

Example: $9y'' + 6y' + y = 0$

Characteristic equation: $9\lambda^2 + 6\lambda + 1 = 0$

$(3\lambda+1)^2 = 0$

$\lambda = -\frac{1}{3}$ (repeated)

Fundamental solutions: $e^{-\frac{1}{3}t}$, $te^{-\frac{1}{3}t}$

General solution: $c_1 e^{-\frac{1}{3}t} + c_2 t e^{-\frac{1}{3}t}$

Non-real complex roots $\lambda = \mu \pm \nu i$:

Fundamental solutions:

$$e^{\mu t}\cos(\nu t), \quad e^{\mu t}\sin(\nu t)$$

General solution:

$$c_1 e^{\mu t}\cos(\nu t) + c_2 e^{\mu t}\sin(\nu t)$$

Example: $y'' - 2y' + 5y = 0$

Characteristic equation: $\lambda^2 - 2\lambda + 5 = 0$

$$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

Fundamental solutions: $e^{t}\cos(2t)$, $e^{t}\sin(2t)$

General solution: $c_1 e^{t}\cos(2t) + c_2 e^{t}\sin(2t)$
